Appl Phys A
DOI 10.1007/s00339-010-6227-6

Applied Physics A
Materials Science & Processing

Analytical formulation of the radiation field of printed antennas in the presence of artificial magnetic superstrates

Hussein Attia · Leila Yousefi · Omar Siddiqui · Omar M. Ramahi

Received: 16 January 2010 / Accepted: 3 December 2010

Abstract In this paper, the cavity model of a microstrip patch antenna in conjunction with the reciprocity theorem is used to develop a fast analytical solution for the radiation field of a microstrip patch antenna loaded with a novel artificial magnetic superstrate and to investigate the effect of the engineered superstrate layer on the directivity and radiation pattern of the printed patch antenna.

1 Introduction

Using numerical methods to analyze periodic structures (metamaterials) is an expensive computational task, which requires huge computer resources [1]. Therefore; in these problems, numerical methods may not be used in the design, and optimization process which needs several iterations, and should be used only to validate the final design. In this paper the cavity model of a microstrip patch antenna [2] and the reciprocity theorem are used to develop an analytical solution for the radiation field of a microstrip patch antenna loaded with a novel artificial magnetic superstrate. To investigate the accuracy of the proposed analytical solution, the analytical results for the antenna loaded with the superstrate have been compared with numerical results obtained from the full-wave electromagnetic simulation tool CST "Computer Simulation Technology" for a specific example. In this example, the antenna is designed to operate at 2.2 GHz at which the artificial magnetic superstrate has an effective permeability of 15. The modified split ring resonator (MSRR) inclusions are used in the design of the artificial magnetic superstrate [3]. Good agreement between the analytical and numerical results is achieved.

H. Attia · L. Yousefi · O. Siddiqui · O.M. Ramahi (✉)
Electrical and Computer Engineering Department, University of Waterloo, Waterloo, Ontario, N2L3G1, Canada
e-mail: oramahi@ece.uwaterloo.ca

2 Analytical formulation for the antenna's far field

In this section, the reciprocity theorem and the cavity model [2] of a microstrip patch antenna are used to analyze the radiation properties of a microstrip patch antenna covered with an artificial magnetic material (metamaterial) acting as a superstrate to enhance the directivity of the antenna. The patch antenna is printed on a grounded substrate of thickness h having relative permeability and permittivity of μ_1, ε_1. At distance d from the substrate is the superstrate layer of thickness B having relative permeability and permittivity of μ_2, ε_2. On top of the superstrate is free space, with total permeability and permittivity of μ_o, ε_o (see Fig. 1).

According to the reciprocity theorem [4], we need to construct two problems. In the first problem, the original radiating patch at $z = h$ is replaced by two magnetic sources $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ (using the cavity model [2]) radiating a far electric field of $\boldsymbol{E}_{1,2}$ at the observation point of $P_{(r,\theta,\text{Ø})}$, and in the second problem we will have a fictitious far dipole (reciprocity source) of J_2 at the same observation point having its far magnetic field $\boldsymbol{H}_2$ at the original patch location at $z = h$. The general integral form of the reciprocity theorem is reduced to

$$\iiint_V [\boldsymbol{M}_{1,2} \cdot \boldsymbol{H}_2]\, dV = -\iiint_V [\boldsymbol{J}_2 \cdot \boldsymbol{E}_{1,2}]\, dV \qquad (1)$$

The reciprocity source is assumed to have a value of $\boldsymbol{J}_2 = \delta(\boldsymbol{r} - \boldsymbol{r}_p)\hat{u}$ where:

$\hat{u} = \hat{\theta}(\hat{\text{Ø}})$ for TM (TE) incident wave

The volume bounded by the microstrip patch and the ground plane can be modeled as a cavity resonator assuming the four walls of this volume to be ideal open circuit (magnetic

Published online: 07 January 2011

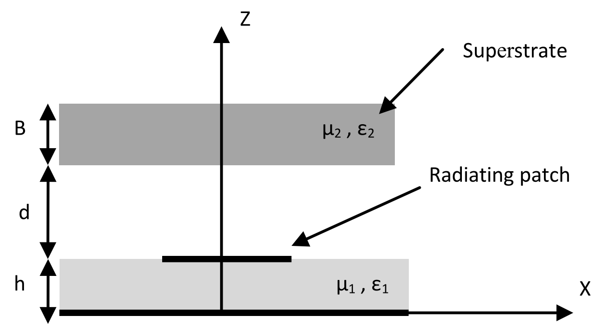


Fig. 1 Microstrip patch antenna covered by a superstrate

walls). The mode of concern here is the dominant transverse magnetic mode (TM_{10}) which presumes a zero value of H_z but a non-zero value of E_z. By using the expression of E_z under ideal magnetic walls boundary condition, one can formulate the equivalent magnetic current in the cavity's apertures as $-\vec{n} \times \vec{E}$. The resultant magnetic currents will be y directed. Hence, (1) reduces to:

$$\boldsymbol{E}_{1,2}(\boldsymbol{r}_p) \cdot \hat{u} = -\iiint_V \left[\boldsymbol{M}_{1,2} \cdot \boldsymbol{H}_2\right] dV \tag{2}$$

It is clear from (2) that the $\boldsymbol{H}_2$ field is determined at the original patch antenna location due to the reciprocity source at $P(r, \theta, \text{Ø})$ in either the $\hat{\theta}$ or $\hat{\text{Ø}}$ direction, when this dipole source is far from the origin. By reciprocity, this $\boldsymbol{H}_2$ is proportional to the required radiated field $\boldsymbol{E}_{1,2}(\boldsymbol{r}_p)$ due to the original patch antenna at $z = h$. The $\boldsymbol{H}_2$ field near the superimposed structure due to this reciprocity source is basically a plane wave, and therefore can be found by modeling each layer as a transmission line (see Fig. 2) having a characteristic impedance and propagation constant which depends on the incident angle of θ [4].

$$n_1^2 = \mu_1\varepsilon_1, \qquad n_2^2 = \mu_2\varepsilon_2, \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$
$$K_0 = \omega\sqrt{\mu_0\varepsilon_0} \tag{3}$$

$$n_1(\theta) = n_1\cos(\theta_{t2}) = \sqrt{n_1^2 - \sin^2(\theta)}$$
$$n_2(\theta) = n_2\cos(\theta_{t1}) = \sqrt{n_2^2 - \sin^2(\theta)} \tag{4}$$

$$\beta_0 = K_{Z0} = K_0\cos(\theta), \qquad \beta_1 = K_{Z1} = K_0 n_1(\theta)$$
$$\beta_2 = K_{Z2} = K_0 n_2(\theta) \tag{5}$$

For TE wave or perpendicular polarization:

$$Z_{C0} = \eta_0\sec(\theta), \qquad Z_{C1} = \frac{\eta_0\mu_1}{n_1(\theta)}, \qquad Z_{C2} = \frac{\eta_0\mu_2}{n_2(\theta)} \tag{6}$$

For TM wave or parallel polarization:

$$Z_{C0} = \eta_0\cos(\theta), \qquad Z_{C1} = \frac{\eta_0 n_1(\theta)}{\varepsilon_1}$$
$$Z_{C2} = \frac{\eta_0 n_2(\theta)}{\varepsilon_2} \tag{7}$$

The electric field of the patch antenna is calculated at the desired frequency using (2), and integrated as follows to calculate the antenna directivity:

$$Directivity(\theta, \Phi) = \frac{4\pi * \cos^2(X)\sin^2(Y)/Y^2(\sin^2(\Phi)\cos^2(\theta)|F(\theta)|^2 + \cos^2(\Phi)|G(\theta)|^2)}{\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos^2(X)\sin^2(Y)/Y^2(\sin^2(\Phi)\cos^2(\theta)|F(\theta)|^2 + \cos^2(\Phi)|G(\theta)|^2)\sin(\theta)\,d\theta\,d\Phi}$$
$$Y = \frac{K_0 W}{2}\sin(\theta)\sin(\Phi), \qquad X = \frac{K_0 L}{2}\sin(\theta)\cos(\Phi) \tag{8}$$

where W and L are the patch dimensions.

3 Artificial magnetic structure as a superstrate for planar antennas

Here we use the method explained in the previous section, to analyze an antenna loaded with an artificial magnetic superstrate (see Fig. 3). The patch antenna used here has dimensions of 36 mm × 36 mm, and is printed on a substrate of Rogers RO4350 having a relative permittivity of 3.48, and a thickness of 0.762 mm. This antenna is designed to operate at the frequency band of 2190–2210 MHz (UMTS) at which the magnetic superstrate has an effective permeability of about 15 (real part) and a magnetic loss tangent of 0.11 (see Fig. 4). For more details about the structure of the artificial magnetic superstrate, and its dimensions, please refer to our previous work in [3].

Since the MSRRs are aligned in the XY plane, the resultant effective enhanced permeability is provided only in the z direction. Any incident magnetic field in the x or y direction will not couple to the MSRR inclusion resulting in a permeability equal to that of free-space in those directions. Hence, the engineered material composed of the MSRR inclusions will experience the anisotropic permeability tensor

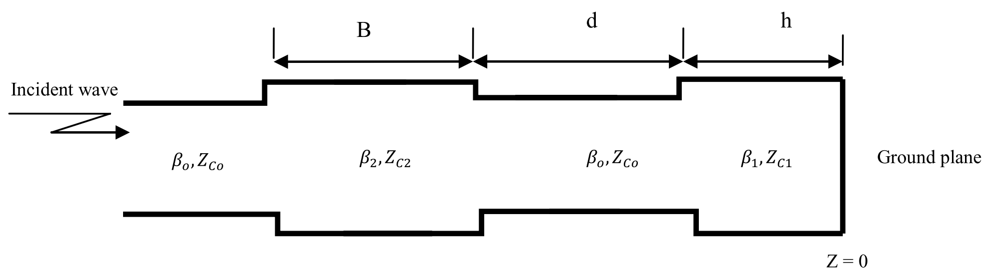


Fig. 2 Transmission line equivalent model of the structure of Fig. 1

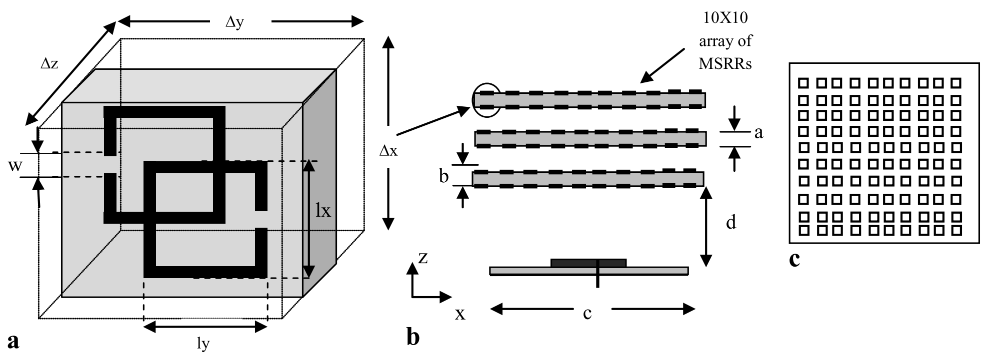


Fig. 3 Geometry of a patch antenna covered by an engineered magnetic superstrate. (**a**) MSRR unit cell. (**b**) Side view. (**c**) Top view ($a = 0.762$ mm, $b = 2$ mm, $c = 85$ mm and $d = 12$ mm)

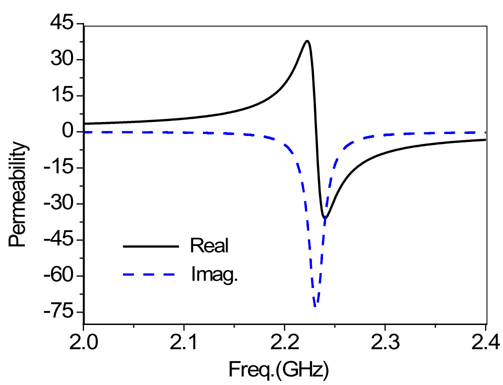


Fig. 4 Analytically calculated relative permeability of the MSRR inclusions

of

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu_{r\_\mathrm{eff}} \end{bmatrix} \tag{9}$$

The analytically calculated effective relative permeability is shown in Fig. 4 (refer to [3] for formulas used in this calculation). Inter-cell capacitors are formed in the gap regions between the metallic inclusions (unit cells) due to the incident x-directed electric field. Therefore, an effective x and y-directed permittivity is provided by the stored electrical energy in those inter-cell capacitors. In case of a z-directed electric field, the metamaterials superstrate will experience an effective permittivity equal to that of its host dielectric as the electric field would be perpendicular to the plane of the unit cell. Therefore, the artificial magnetic material composed of the MSRRs inclusions will experience anisotropic electric permittivity of

$$\bar{\bar{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_{r\_\mathrm{eff}} & 0 & 0 \\ 0 & \varepsilon_{r\_\mathrm{eff}} & 0 \\ 0 & 0 & \varepsilon_{r\_\mathrm{diel}} \end{bmatrix} \tag{10}$$

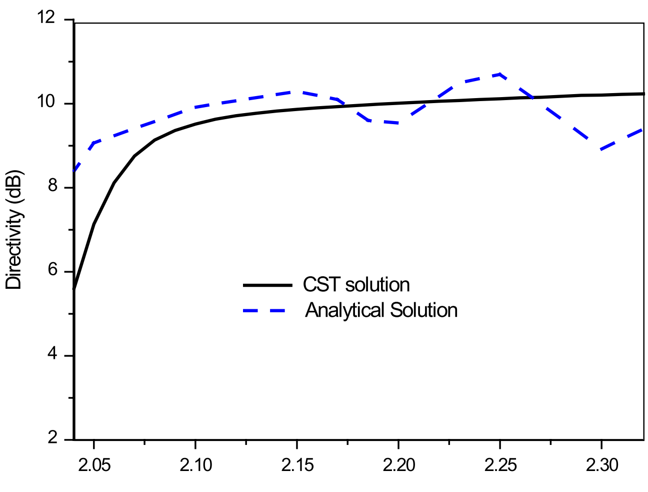
For the formula of $\varepsilon_{r\_\mathrm{eff}}$, please refer to [3]. According to the above formulas, the effective relative permittivity of the designed structure in the x, and y directions would be equal to 5.62.

Substituting the value of permittivity, and permeability from (9), (10) in the equations presented in Sect. 2, the electric field of the patch antenna is calculated at different frequencies and integrated to calculate the antenna directivity using (8).

Figure 5 shows a comparison between the analytical and numerical results of the antenna directivity and radiation pattern. A good agreement is observed between both methods. This agreement verifies the accuracy of the proposed analytical model.

4 Conclusions

A fast accurate analytical technique has been presented for the problem of a microstrip patch antenna covered with an artificial magnetic superstrate. The analytical solution is based on the cavity model of a microstrip patch antenna and the reciprocity theorem. A good agreement has been found between this analytical solution and the results obtained from the commercial electromagnetic software CST.



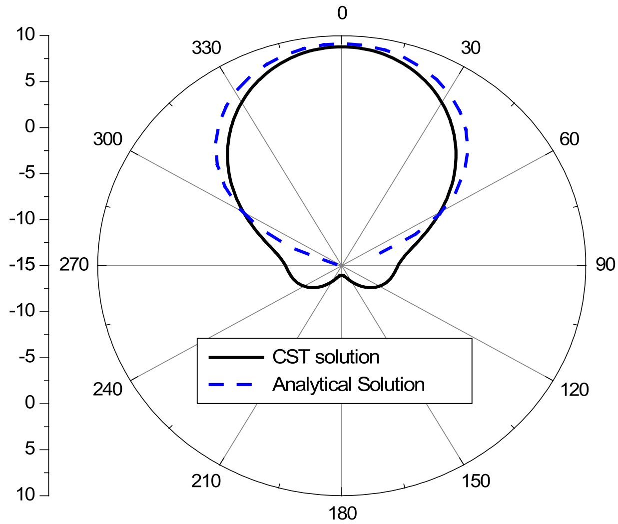


Fig. 5 (**a**) The directivity of the microstrip antenna covered with the artificial magnetic superstrate calculated using CST and the analytical solution. (**b**) The radiation pattern (E-plane, $\text{Ø} = 0$) at 2.2 GHz of the patch antenna covered with the engineered magnetic superstrate

Acknowledgement This work has been supported by RIM, *Research In Motion* and the Ministry of Higher Education, Egypt.

References

1. Y.J. Lee, J. Yeo, R. Mittra, W.S. Park, Application of electromagnetic bandgap (EBG) superstrates with controllable defects for a class of patch antennas as spatial angular filters. IEEE Trans. Antennas Propag. **53**(1), 224–235 (2005)
2. K. Carver, J. Mink, Microstrip antenna technology. IEEE Trans. Antennas Propag. **29**(1), 2–24 (1981)
3. H. Attia, L. Yousefi, M.M. Bait-Suwailam, M.S. Boybay, O.M. Ramahi, Enhanced-gain microstrip antenna using engineered magnetic superstrates. IEEE Antennas Wirel. Propag. Lett. **8**, 1198–1201 (2009)
4. D.R. Jackson, N.G. Alexopoulos, Gain enhancement methods for printed circuit antennas. IEEE Trans. Antennas Propag. **33**(9), 976–987 (1985)